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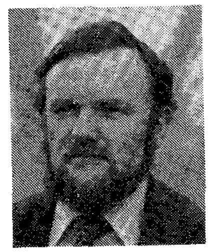

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# Millimeter-Wave Performance of Shielded Slot-Lines

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*Abstract*—**The high frequency characteristics of shielded slot-lines are investigated using a modified Wiener–Hopf technique. The analysis includes the case of uniaxially anisotropic substrates when the principal axis is directed normal to the substrate. The obtained solution is especially useful at higher frequencies where other methods tend to be less effective. Numerical results are given for lines on high dielectric constant substrates over the full usable frequency range. It is shown that lines can be used as transmission elements up to a certain limiting frequency beyond which they will radiate. Physical aspects of the propagation in slot-lines are discussed and the effect of the shields on the properties of the guided modes is explained.**

## I. Introduction

SLOT-LINES are attracting increasing interest as a transmission element in integrated circuits at higher microwave and millimeter-wave frequencies. Since their introduction by Cohn and others [1], [2], they have been investigated by a number of authors. At lower microwave frequencies, slot-lines were not much used because of the availability of other efficient and easy-to-calculate transmission lines. At higher frequencies, however, as microstrip lines and waveguides become inadequate for some applications and alternatives are searched, the slot-line becomes a promising element in many respects. The waveguide-housed counterparts, unilateral fin-lines, have been successfully used for many millimeter-wave applications. The slot-line as an open structure was felt to be susceptible to excessive radiation at the discontinuities and to cross-coupling. At higher frequencies, shielded slot-lines may be as efficient as fin-lines and are much easier to integrate. One of the important factors interfering with the application of slot-lines is the lack of data on its performance at higher frequencies. Being inherently non-TEM line, it is much more difficult to calculate, compared to microstrip lines or fin-lines. The latter are frequently treated as ridged rectangular waveguides.

The results of [1], [2] are based on waveguide iris impedance concepts and are sufficiently accurate for narrow slot-lines. Various approaches were suggested for the full-wave treatment of these lines and a fair amount of data is now available [2]–[6]. However, the basic features of prop-

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agation in these lines, as well as their high-frequency limitations, do not seem to be fully clarified yet. Moreover, slot-lines on anisotropic substrates were not considered in any detail in literature. Slot-lines on anisotropic substrates have potential application as millimeter-wave transmission media and in integrated optics.

The following analysis of shielded slot-lines is based on the functional equations approach which was applied to bilateral fin-lines and to microstrip lines on anisotropic substrates [7], [8]. The main advantage of the method is its superior numerical efficiency, especially at high frequencies, and the physically identifiable quantities appearing in the expressions for fields and currents, allowing a deeper insight into the physics of the problem.

## II. Formulation of the Problem

The configuration of the shielded slot-line under consideration is shown on Fig. 1. The slot-line with gap-width $W$ and semi-infinite thin conductors lies in the plane $z=0$ over a dielectric substrate which is assumed to be, in general, uniaxially anisotropic with the principal axis in the direction of $z$ perpendicular to the plane of the substrate. The thickness of the substrate is taken to be $2d$ and the upper shield is located at a distance $d_1$ from the plane of the line, while the lower shield is at distance $d_2$ from the lower side of the substrate. The cartesian axes $x$, $y$, and $z$ are located as shown. Shields and slot-line are assumed to be perfect conductors. The tensor permittivity of the substrate material is written

$$\hat{\varepsilon}=\varepsilon_0\begin{pmatrix}\varepsilon_t & 0 & 0\\ 0 & \varepsilon_t & 0\\ 0 & 0 & \varepsilon_z\end{pmatrix}$$

and the magnetic permeability is $\mu_0\mu_r$. $\varepsilon_0$ and $\mu_0$ are the permittivity and permeability of free space, respectively. The dependence of the fields and currents on the time $t$ and the longitudinal coordinate $y$ is given by $e^{-i(\omega t-\gamma y)}$, where $\omega$ is the angular frequency and $\gamma$ is a real propagation constant. Fields and currents will be represented through their Fourier transforms

$$f(x,z)=\int_{-\infty}^{+\infty}\tilde{f}(\alpha,z)e^{-i\alpha x}d\alpha$$

where $f$ is any of the field or current components, $\tilde{f}$ is its Fourier transform, and $\alpha$ is a complex parameter. Field components can be represented through $\tilde{E}_z$ and $\tilde{H}_z$ in the dielectric regions I, II, and III

$$\frac{i}{\kappa^2}\frac{\partial\tilde{E}_z}{\partial z}=-\alpha\tilde{E}_x+\gamma\tilde{E}_y$$
$$-\omega\mu_0\mu_r\tilde{H}_z=\gamma\tilde{E}_x+\alpha\tilde{E}_y \tag{1}$$
$$i\frac{\partial\tilde{H}_z}{\partial z}=-\alpha\tilde{H}_x+\gamma\tilde{H}_y$$
$$\omega\varepsilon_0\varepsilon_z\tilde{E}_z=\gamma\tilde{H}_x+\alpha\tilde{H}_y. \tag{2}$$

If the functions $\tilde{E}_z$ and $\tilde{H}_z$ are known, all other field components can be determined from (1) and (2). Functions $\tilde{E}_z$ and $\tilde{H}_z$ satisfy the one dimensional wave equations

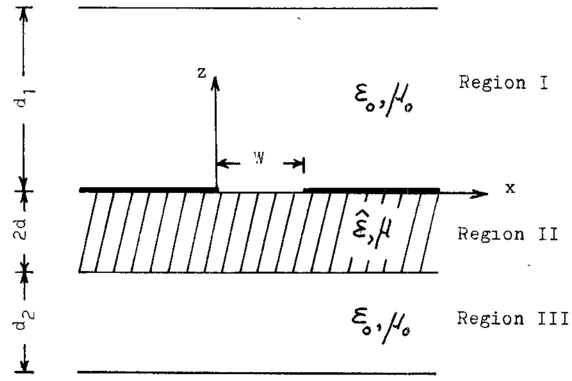


Fig. 1. Shielded slot-line configuration.

$$\left[\frac{d^2}{dz^2}-\kappa^2\left(\alpha^2+\gamma^2-k_0^2\varepsilon_z\mu_r\right)\right]\tilde{E}_z=0$$
$$\left[\frac{d^2}{dz^2}-\left(\alpha^2+\gamma^2-k_0^2\varepsilon_t\mu_r\right)\right]\tilde{H}_z=0 \tag{3}$$

where

$$\kappa^2=\varepsilon_t/\varepsilon_z \qquad k_0^2=\omega^2\varepsilon_0\mu_0.$$

### A. Boundary Conditions

Continuity relations for the electric and magnetic fields on the interfaces between different media together with relations (1) and (2) lead to the following boundary conditions for $\tilde{E}_z$ and $\tilde{H}_z$ and their derivatives.

1) On the shields $\tilde{E}_x=\tilde{E}_y=0$; therefore

$$\left.\frac{\partial\tilde{E}_z}{\partial z}\right]_{z=d_1}=\left.\frac{\partial\tilde{E}_z}{\partial z}\right]_{z=-2d-d_1}=0$$
$$\tilde{H}_z(\alpha,d_1)=\tilde{H}_z(\alpha,-2d-d_1)=0. \tag{4}$$

2) On the interfaces $z=0$ and $z=-2d$, the tangential components of the electric field are continuous. For $\tilde{E}_z$ and $\tilde{H}_z$ we have

$$\left.\frac{\partial\tilde{E}_z}{\partial z}\right]_{z=+0}=\frac{1}{\kappa^2}\left.\frac{\partial\tilde{E}_z}{\partial z}\right]_{z=-0}$$
$$\frac{1}{\kappa^2}\left.\frac{\partial\tilde{E}_z}{\partial z}\right]_{z=-2d+0}=\left.\frac{\partial\tilde{E}_z}{\partial z}\right]_{z=-2d-0} \tag{5}$$
$$\tilde{H}_z(\alpha,+0)=\mu_r\tilde{H}_z(\alpha,-0)$$
$$\mu_r\tilde{H}_z(\alpha,-2d+0)=\tilde{H}_z(\alpha,-2d-0). \tag{6}$$

3) On the interface $z=-2d$, $\tilde{H}_x$ and $\tilde{H}_y$ are continuous, i.e.,

$$\left.\frac{\partial\tilde{H}_z}{\partial z}\right]_{z=-2d+0}=\left.\frac{\partial\tilde{H}_z}{\partial z}\right]_{z=-2d-0} \tag{7}$$
$$\varepsilon_z\tilde{E}_z(\alpha,-2d+0)=\tilde{E}_z(\alpha,-2d-0). \tag{8}$$

4) Due to the presence of surface currents, $\tilde{J}_x(\alpha)$ and $\tilde{J}_y(\alpha)$ flowing on the slot-line conductors, fields $\tilde{H}_x$ and $\tilde{H}_y$ are discontinuous and so are $\varepsilon_z$, $\tilde{E}_z$, and $\partial\tilde{H}_z/\partial z$ on the plane $z=0$. This leads to the following boundary condi-

tions on the plane of the conductors:

$$\begin{aligned}\omega\varepsilon_0\left[\tilde{E}_z(\alpha,+0)-\varepsilon_z\tilde{E}_z(\alpha,-0)\right] &= -\alpha\tilde{J}_x(\alpha)-\gamma\tilde{J}_y(\alpha)\\ &= U_1 - i\left\{\frac{\partial\tilde{H}_z}{\partial z}\right]_{z=+0} - \left.\frac{\partial\tilde{H}_z}{\partial z}\right]_{z=-0}\right\}\\ &= \gamma\tilde{J}_x(\alpha)+\alpha\tilde{J}_y(\alpha)=U_2 \end{aligned}\tag{9}$$

where $U_1$ and $U_2$ are introduced as linear combinations of $\tilde{J}_x$ and $\tilde{J}_y$ in the Fourier transform domain.

Expressions for $\tilde{E}_z(\alpha, z)$ and $\tilde{H}_z(\alpha, z)$ in regions I, II, and III taking into account boundary conditions (4), (5), (6), and (7) can be written as follows.

*Region I:*

$$\tilde{E}_z(\alpha,z) = -A\frac{\cosh R_0(d_1-z)}{R_0\sinh R_0 d_1}$$

$$\tilde{H}_z(\alpha,z) = B\frac{\sinh R_0(d_1-z)}{\sinh R_0 d_1}.$$

*Region II:*

$$\tilde{E}_z(\alpha,z) = -C\kappa\frac{\cosh R_z z}{R_z\sinh 2R_z d}+\kappa A\frac{\cosh R_z(2d+z)}{R_z\sinh 2R_z d}$$

$$\tilde{H}_z(\alpha,z) = -\frac{D}{\mu_r}\frac{\sinh R_t z}{\sinh 2R_t d}+\frac{B}{\mu_r}\frac{\sinh R_t(2d+z)}{\sinh 2R_t d}.$$

*Region III:*

$$\tilde{E}_z(\alpha,z) = C\frac{\cosh R_0(2d+d_2+z)}{R_0\sinh R_0 d_2}$$

$$\tilde{H}_z(\alpha,z) = D\frac{\sinh R_0(2d+d_2+z)}{\sinh R_0 d_2}.$$

$A, B, C, D$ are some function of the parameter $\alpha$

$$R_0=\sqrt{\alpha^2+\gamma^2-k_0^2}$$

$$R_z=\sqrt{\alpha^2+\gamma^2-k_0^2\mu_r\varepsilon_z}$$

$$R_t=\sqrt{\alpha^2+\gamma^2-k_0^2\mu_r\varepsilon_t}.$$

Applying conditions (8), we obtain the following relations between $A$ and $B$ and $C$ and $D$:

$$D\left(R_0\coth R_0 d_2+\frac{R_t}{\mu_r}\coth 2R_t d\right)=\frac{R_t}{\mu_r}\frac{1}{\sinh 2R_t d}B$$

$$C\left(\frac{\coth R_0 d_2}{R_0}+\varepsilon_e\frac{\coth 2R_z d_e}{R_z}\right)=\frac{\varepsilon_e}{R_z}\frac{1}{\sinh 2R_z d_e}A$$

where

$$\varepsilon_e=\sqrt{\varepsilon_t\varepsilon_z}\qquad d_e=d\kappa.$$

Finally, application of (9) and using the relations between $A$, $B$, $C$, and $D$ gives $A$ and $B$ in terms of $U_1$ and $U_2$

$$\begin{aligned}-\omega\mu_0 A\chi_1(\alpha)&=U_1(\alpha)\\ iB\chi_2(\alpha)&=U_2(\alpha)\end{aligned}\tag{10}$$

where

$$\chi_1(\alpha)=\frac{\coth R_0 d_1}{R_0}+\varepsilon_e\frac{\coth 2R_z de}{R_z}-\left(\frac{\varepsilon_e}{R_z\sinh 2R_z d_e}\right)^2\cdot\frac{1}{\dfrac{\coth R_0 d_2}{R_0}+\varepsilon_e\dfrac{\coth 2R_z d_e}{R_z}}$$

$$\chi_2(\alpha)=R_0\coth R_0 d_1+\frac{R_t}{\mu_r}\coth 2R_t d-\left(\frac{R_t}{\mu_r\sinh 2R_t d}\right)^2\cdot\frac{1}{R_0\coth R_0 d_2+\dfrac{R_t}{\mu_r}\coth 2R_t}.$$

Introducing the functions

$$F_1(\alpha)=\left.\frac{i}{\kappa^2}\frac{\partial\tilde{E}_z}{\partial z}\right]_{z=0}=-\alpha\tilde{E}_x(\alpha,0)+\gamma\tilde{E}_y(\alpha,0)=iA$$

$$\begin{aligned}F_2(\alpha)&=-\omega\mu_0\mu_r\tilde{H}_z(\alpha,0)=\gamma\tilde{E}_x(\alpha,0)+\alpha\tilde{E}_y(\alpha,0)\\ &=-\omega\mu_0 B.\end{aligned}$$

The relation between $F_1$ and $F_2$ and $U_1$ and $U_2$ can now be written

$$\begin{aligned}i\omega\varepsilon_0\chi_1(\alpha)F_1(\alpha)&=U_1(\alpha)\\ \chi_2(\alpha)F_2(\alpha)&=i\omega\mu_0 U_2(\alpha).\end{aligned}\tag{11}$$

Similar relations have been used for the analysis of shielded microstrip lines on anisotropic substrates [8]. Functions for the microstrip lines are the limiting cases of (10) when $d_2=0$.

Equations (11) represent the independent excitation of LSM- and LSE-modes in the structure by the currents flowing on the conductors of the slot-line. Thus, if $U_2=0$ then $B=0$, $F_2=0$, i.e., $\tilde{H}_z=0$ and the field is purely LSM. Similarly, when $U_1=0$, the field is LSE and $\tilde{E}_z=0$. $\chi_1$ and $\chi_2$ are the inverse Green's functions of these field types. In the shielded structure when both $d_1$ and $d_2$ are finite, $\chi_1$ and $\chi_2$ are meromorphic even functions of $\alpha$, having pole singularities at points $\pm\alpha_n$ and $\pm\beta_n$, and zeroes at $\pm\nu_n$ and $\pm\sigma_n$ (the imaginary parts of $\alpha_n$, $\beta_n$, $\nu_n$, and $\sigma_n$ are assumed to be positive), respectively. The propagation constants of LSM- and LSE-modes in the dielectric slab loaded structure are the zeros of $\chi_1$ and $\chi_2$, when $\gamma=0$ and will be denoted by $\bar{\nu}_n$ and $\bar{\sigma}_n$. Due to the fact that $\alpha$ and $\nu$ are always combined in the arguments of $\chi_1$ and $\chi_2$ in the form $\alpha^2+\gamma^2$, the zeros for arbitrary $\gamma$ are related to $\bar{\nu}_n$ and $\bar{\sigma}_n$ by

$$\nu_n^2=\bar{\nu}_n^2-\gamma^2\qquad\sigma_n^2=\bar{\sigma}_n^2-\gamma^2.$$

When $\gamma=0$, these modes propagate laterally in the directions of $\pm x$. When $\gamma\neq 0$, they have a longitudinal component of propagation constant given by $\gamma$ and a transverse component given by $\nu_n$ and $\sigma_n$. In the slot-line, the field in the gap region can be represented as a combination of these modes.

Similarly, the poles of $\chi_1$ and $\chi_2$ are the roots of the equations

$$\left.\begin{aligned}\frac{\coth R_0 d_2}{R_0}+\varepsilon_e\frac{\coth 2R_z d_e}{R_z}=0\\ R_0\tanh R_0 d_1=0\end{aligned}\right\}\text{for } \alpha_n$$

$$\left.\begin{aligned}R_0\coth R_0 d_2+\frac{R_t}{\mu_r}\coth 2R_t d=0\\ \frac{1}{R_0}\tanh R_0 d_1=0\end{aligned}\right\}\text{for } \beta_n.$$

When $\gamma=0$, they will be denoted by $\bar{\alpha}_n$ and $\bar{\beta}_n$ and are related to $\alpha_n$ and $\beta_n$ by

$$\alpha_n^2=\bar{\alpha}_n^2-\gamma^2 \qquad \beta_n^2=\bar{\beta}_n^2-\gamma^2.$$

The roots and poles $\bar{\nu}_n$, $\bar{\sigma}_n$, $\bar{\alpha}_n$, and $\bar{\beta}_n$ are functions of frequency and the parameters of the line, but not of $\gamma$. The poles of $\chi_1$ and $\chi_2$ coincide with the propagation constants of the waveguide modes in the regions outside the slot, between the planar line conductors, and the shields. The fields in these regions can evidently be represented as a combination of these modes.

The slot-line problem can finally be formulated in the Fourier transform domain as follows. It is required to determine the analytic functions $F_1$, $F_2$, $U_1$, and $U_2$ satisfying the relations (11) and are such that the tangential components of the fields and the surface currents satisfy the boundary conditions on the gap and line conductors. Tangential electric field must vanish on the planar conductors while the currents vanish on the gap. Both electric field and current components must satisfy the edge conditions [9].

## III. Guided Wave Propagation in Slot-Lines

To determine the fields and propagation constants of the modes guided by the slot-line, the method developed in [7] will be followed. Boundary and edge conditions on the tangential field and current components are written as follows.

1) On the planar conductors

$$E_x(x,0)=0 \qquad E_y(x,0)=0, \qquad \text{for } x<0 \qquad x>W.$$

2) On the gap

$$J_x(x)=0 \qquad J_y(x)=0, \qquad \text{for } 0<x<W.$$

3) At the edge $x\to 0$ we have

$$E_x(x,0)\sim x^{-1/2} \qquad E_y(x,0)\sim x^{1/2}$$
$$J_x(x)\sim x^{1/2} \qquad J_y(x)\sim x^{-1/2}.$$

The same conditions apply on the other edge $x\to W$. Boundary conditions 1, 2, and 3 will reflect on the properties of the functions $F_1$, $F_2$, $U_1$, and $U_2$ as follows.

1) $F_1$ and $F_2$ are entire functions which grow at infinity as $e^{i\alpha W}$.

2) Functions $U_1$ and $U_2$ can be represented in the form

$$U_1(\alpha)=U_1^-(\alpha)-e^{i\alpha W}U_1^-(-\alpha)$$
$$U_2(\alpha)=U_2^-(\alpha)+e^{i\alpha W}U_2^-(-\alpha)$$

where $U_1^-$ and $U_2^-$ are regular on the lower half-plane $\operatorname{Im}\alpha<0$. These expressions take into account the symmetry of the current components $J_x$ and $J_y$ with respect to the gap center.

3) The asymptotic behavior of $F_1$, $F_2$, $U_1$, and $U_2$ when $\alpha\to\infty$ is determined by the singularities of the fileds and currents on the gap edges and can be shown to be

$$F_1(\alpha)\sim\alpha^{1/2} \qquad F_2(\alpha)\sim\alpha^{-1/2}$$
$$U_1(\alpha)\sim\alpha^{-1/2} \qquad U_2(\alpha)\sim\alpha^{1/2}.$$

The slot-line problem thus reduces to the solutions of the functional equations

$$i\omega\varepsilon_0\chi_1(\alpha)F_1(\alpha)=U_1^-(\alpha)-e^{i\alpha W}U_1^-(-\alpha)$$
$$\frac{1}{i\omega\mu_0}\chi_2(\alpha)F_2(\alpha)=U_2^-(\alpha)+e^{i\alpha W}U_2^-(-\alpha). \tag{12}$$

Equations (12) can be reduced to systems of simultaneous algebraic equations through the application of modified Wiener–Hopf technique [7], [9]. Representing functions $\chi_1$ and $\chi_2$ as products of functions regular and having no roots in the upper and lower half planes

$$\chi_1(\alpha)=\chi_1^+(\alpha)\chi_1^-(\alpha) \qquad \chi_2(\alpha)=\chi_2^+(\alpha)\chi_2^-(\alpha)$$

then dividing (12) by $\chi_1^-$ and $\chi_2^-$ and separating the minus and plus terms of the resulting equations we obtain

$$\frac{1}{\chi_1^-}U_1^-(\alpha)-\left[e^{i\alpha W}\frac{U_1^-(-\alpha)}{\chi_1^-}\right]^- =\frac{1}{\chi_1^-}U_1^-(\alpha)-\sum_n e^{i\nu_n W}\frac{U_1^-(-\nu_n)}{\chi_1^-(\nu_n)(\alpha-\nu_n)}=P$$

$$\frac{1}{\chi_2^-}U_2^-(\alpha)+\left[e^{i\alpha W}\frac{U_2^-(-\alpha)}{\chi_2^-}\right]^- =\frac{1}{\chi_2^-}U_2^-(\alpha)+\sum_n e^{i\sigma_n W}\frac{U_2^-(-\sigma_n)}{\chi_2^-(\sigma_n)(\alpha-\sigma_n)}=Q \tag{13}$$

where $P$ and $Q$ are some constants. Equations (13) account for the asymptotic behavior of the functions and thus satisfy the edge conditions. The unknown coefficients $U_1^-(-\nu_n)$ and $U_2^-(-\sigma_n)$ can be determined by setting $\alpha=-\nu_n$ in the first equation and $\alpha=-\sigma_n$ in the second, arriving at the following sets of algebraic equations:

$$A_n+\sum_{m=1}^{\infty}\frac{\xi_m}{\nu_m+\nu_n}A_m=1, \qquad n=1,2,\cdots$$
$$B_n+\sum_{m=1}^{\infty}\frac{\zeta_m}{\sigma_m+\sigma_n}B_m=1, \qquad n=1,2,\cdots \tag{14}$$

where

$$PA_n=\frac{U_1^-(-\nu_n)}{\chi_1^-(-\nu_n)} \qquad QB_n=\frac{U_2^-(-\sigma_n)}{\chi_2^-(-\sigma_n)}$$
$$\xi_n=\frac{\chi_1^-(-\nu_n)}{\chi_1^-(\nu_n)}e^{i\gamma_n W} \qquad -\zeta_n=\frac{\chi_2^-(-\sigma_n)}{\chi_2^-(\sigma_n)}e^{i\sigma_n W}.$$

Functions $U_1$ and $U_2$ are expressed in terms of $A$ and $B$ as

$$U_1^-(\alpha)=P\chi_1^-(\alpha)\left\{1+\sum_{m=1}^{\infty}\frac{\xi m}{\alpha-\nu_m}A_m\right\}$$
$$U_2^-(\alpha)=Q\chi_2^-(\alpha)\left\{1+\sum_{m=1}^{\infty}\frac{\zeta m}{\alpha-\sigma_m}B_m\right\}. \qquad (15)$$

The propagation constant $\gamma$ is determined from the characteristics of the transforms of $\tilde{E}_x$ and $\tilde{E}_y$. The inverse linear transformation from $F_1$ and $F_2$ to $\tilde{E}_x$ and $\tilde{E}_y$ will introduce pole singularities in $\tilde{E}_x$ and $\tilde{E}_y$ at values of $\alpha=\pm i\gamma$, unless the functions $F_1$ and $F_2$ satisfy the conditions

$$F_1(\pm i\gamma)\pm iF_2(\pm i\gamma)=0$$

which can be shown to be equivalent to

$$U_1^-(\pm i\gamma)\pm iU_2^-(\pm i\gamma)=0. \qquad (16)$$

Equation (16) represents a set of homogeneous equations for the determination of the unknown constants $P$ and $Q$. $\gamma$ can be determined as the root of the determinant.

Defining the impedance of the slot-line as the ratio of the integral of the $x$-component of the electric field over the gap to the total longitudinal current flowing on one of the line conductors, it can be readily calculated through the $F$ and $U$ functions. Thus

$$U_1^-(0)=\gamma\tilde{J}_y^-(0)=\frac{\gamma}{2\pi}\int_{-\infty}^{0}J_y(x)\,dx=\frac{\gamma}{2\pi}I$$
$$F_2(0)=\gamma\tilde{E}_x(0)=\frac{\gamma}{2\pi}\int_{0}^{W}E_x(x,0)\,dx=\frac{\gamma}{2\pi}V$$

where $V$ is the quasi-static gap voltage and $I$ is the total longitudinal current on the conductor $x<0$. The impedance is therefore given by

$$Z_0=\frac{F_2(0)}{U_1^-(0)}=\frac{2i\omega\mu_0U_2^-(0)}{\chi_2(0)U_1^-(0)}.$$

## IV. Characteristics of Propagation in Slot-Lines and Numerical Results

A considerable amount of information can be gained about the general characteristics of propagation in slot-lines by studying the zero-pole behavior of inverse Green's functions. We notice that although the summations in (15) have poles at the points $\alpha=\nu_n$ and $\alpha=\sigma_n$, functions $U_1^-$ and $U_2^-$ are regular at these points since the zeroes of $\chi_1^-$ and $\chi_2^-$ will cancel the poles. Functions $U_1^-$ and $U_2^-$ have their poles at the points $\alpha=\alpha_n$ and $\alpha=\beta_n$, respectively, coinciding with the poles of $\chi_1^-$ and $\chi_2^-$. This means that, as expected, the fields in the regions outside the gap $x<0$, $x>W$ is a superposition of waveguide modes having transverse propagation constants $\alpha_n$ and $\beta_n$. For guided (unattenuated) propagation, all these modes should be evanescent, i.e., $\alpha_n$ and $\beta_n$ must be purely imaginary, otherwise power will be radiated in the broadside directions. The properties of $\chi_1$ and $\chi_2$ are such that the squares of the poles and zeros, for $\gamma=0$, are real and can be arranged in the sequences

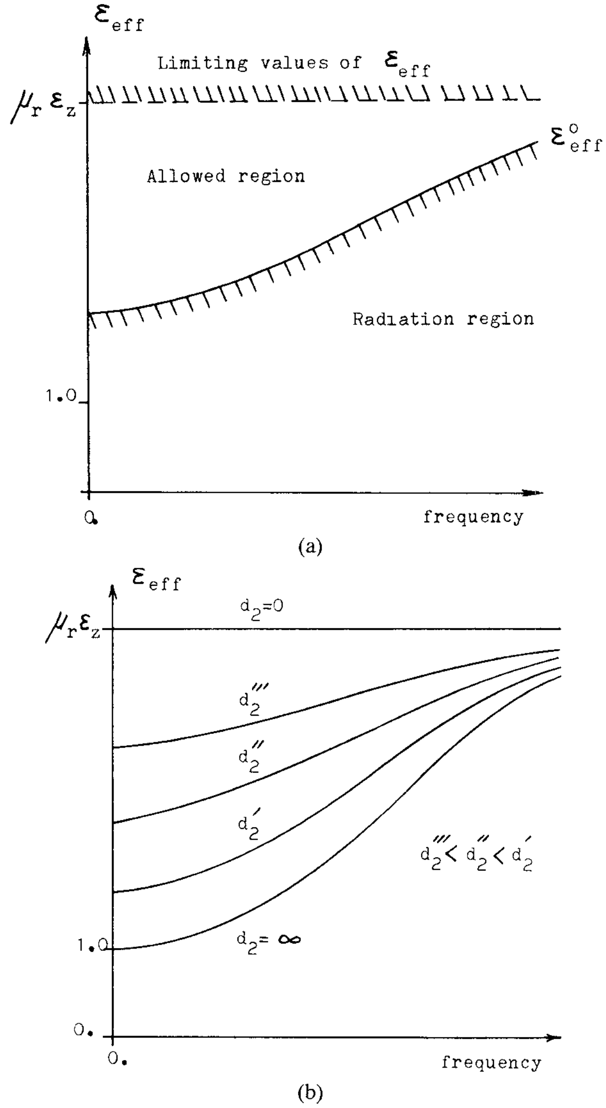


Fig. 2. (a) Region of allowed values of $\varepsilon_{eff}$ for slot-line. (b) Effect of $d_2$ on the characteristics of the lowest order waveguide mode and allowed region.

$$-\infty<\cdots\leqslant\bar{\nu}_2^2\leqslant\bar{\alpha}_1^2\leqslant\bar{\nu}_1^2\leqslant\bar{\alpha}_0^2$$
$$-\infty<\cdots\leqslant\bar{\beta}_2^2\leqslant\bar{\sigma}_2^2\leqslant\bar{\beta}_1^2\leqslant\bar{\sigma}_1^2. \qquad (17)$$

Depending on the the frequency, these poles and roots may be negative or positive. At higher frequencies, larger numbers of these quantities are positive, indicating that the corresponding modes are above their cutoff. In order that all waveguide modes would be evanescent, i.e., $\alpha_n$ and $\beta_n$ are imaginary, the propagation constant $\gamma$ must satisfy the inequalities

$$\gamma^2>\bar{\alpha}_0^2 \qquad \gamma^2>\bar{\beta}_1^2.$$

It can be shown that the dominant mode in the dielectric loaded guide between the slot-line conductors and the lower shield corresponds to the pole $\alpha_0$, i.e.

$$\bar{\alpha}_0^2>\bar{\beta}_1^2>\{\bar{\alpha}_n^2,\bar{\beta}_n^2\}.$$

On the other hand, from physical reasoning, the propagation constant must be smaller than that of plane-wave in the dielectric $k=k_0\sqrt{\mu_r\varepsilon_z}$. Therefore, the possible values of

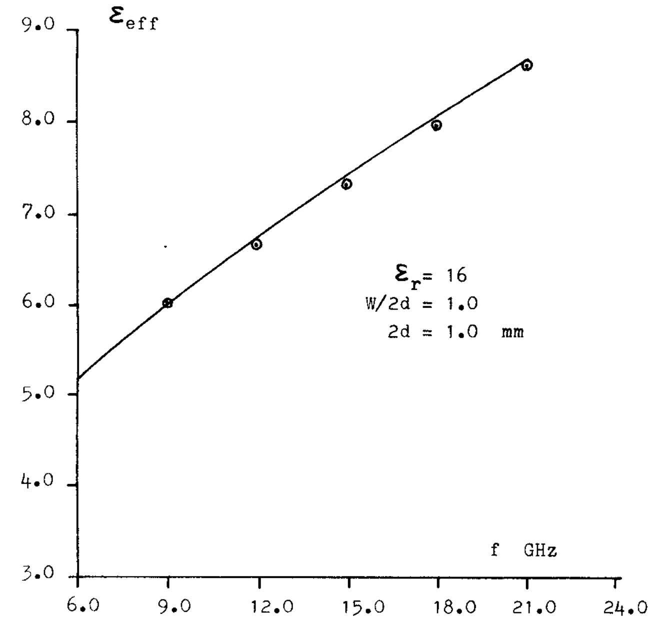


Fig. 3. Open (unshielded) slot-line dispersion characteristics. ——— present theory, ⊙ results of [2] and [4].

$\gamma$ for guided propagation lie in the range

$$\bar{\alpha}_0 < \gamma < k_0\sqrt{\mu_r \varepsilon_z}\,.$$

In terms of the effective dielectric constants we have

$$\varepsilon_{\text{eff}}^0 \leqslant \varepsilon_{\text{eff}} \leqslant \mu_r \varepsilon_z \tag{18}$$

where

$$\varepsilon_{\text{eff}}^0 = \frac{\bar{\alpha}_0^2}{k_0^2} \qquad \varepsilon_{\text{eff}} = \frac{\gamma^2}{k_0^2}.$$

$\varepsilon_{\text{eff}}^0$ represents the effective dielectric constant of the dominant waveguide mode corresponding to $\alpha_0$. Inequality (18) represents the region of the allowed values of $\varepsilon_{\text{eff}}$ for the slot-line on the dispersion diagram $\varepsilon_{\text{eff}} - f$ as shown in Fig. 2(a). Typical dispersion characteristics of the mode $\alpha_0$ for different distances of the lower shield $d_2$ are shown on Fig. 2(b). If the lower shield is absent $d_2 = \infty$, the mode effective dielectric constant is always unity at $f = 0$, the mode propagates with the velocity of light in free space, and tends to $\mu_r \varepsilon_z$ at high frequencies $f \to \infty$. As the distance $d_2$ becomes smaller, the zero-frequency value of $\varepsilon_{\text{eff}}^0$ increases, approaching $\mu_r \varepsilon_z$ as $d_2 \to 0$. Therefore, a conclusion can be drawn, that the propagation in slot-lines is significantly affected by the lower shield. For a given slot width, if the lower shield is too near then the allowed region is so narrow that the guided propagation is impossible. Alternatively, for a fixed value of $d_2$, the slot-line can support guided waves only for gap widths smaller than some maximum value $W_m$, determined by the properties of the substrate and the distances of the shields. The effect of the upper shield is somewhat different but it leads to similar results. It determines the cutoff characteristics and lowers the effective dielectric constant of the slot-line, especially for relatively wide-gap lines.

Relations (17) and (18) also imply that, except possibly $\sigma_1$, all $\nu_n$ and $\sigma_n$ are imaginary in the guided propagation region. The sets of equations in (14) are highly convergent due to the presence of the exponential terms in the summations, especially for relatively wide slots. This explains the numerical efficiency of the method, which is common to most Wiener—Hopf type problems.

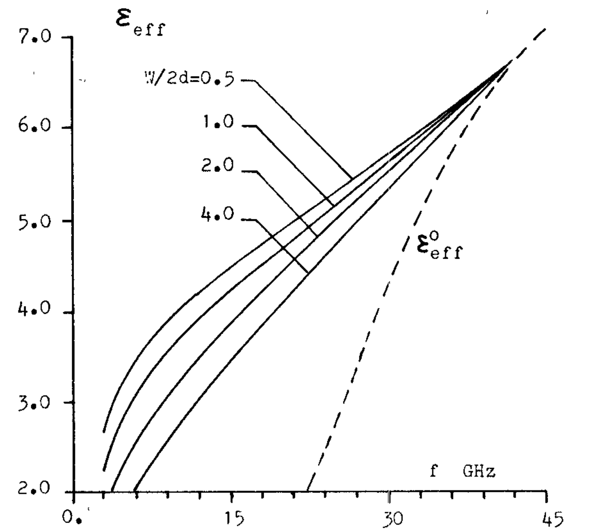


Fig. 4. High-frequency characteristics of shielded slot-line on Alumina. $\varepsilon_r = 9.6$, $\mu_r = 1.0$, $2d = 1.0$ mm, and $d_1/d = d_2/d = 10.0$. - - - - characteristics of lowest order waveguide mode outside the gap region.

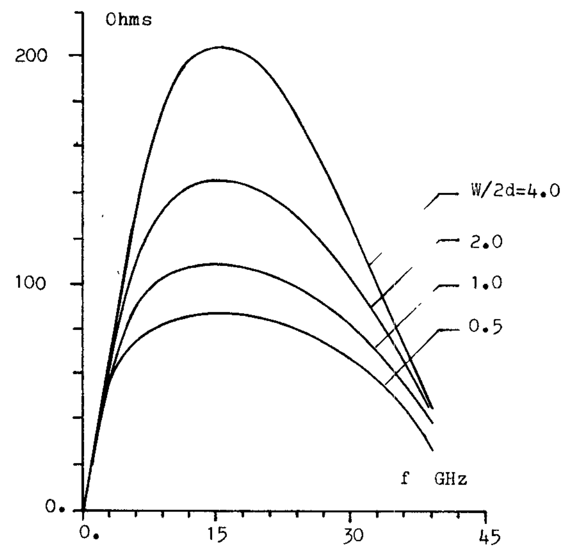


Fig. 5. Impedance of the shielded slot-line on Alumina. $\varepsilon_r = 9.6$, $\mu_r = 1.0$, and $d_1/d = d_2/d = 10.0$.

In order to compare the results of the present theory with published results (which are available only for unshielded lines), the dispersion characteristics of the slot-line on a substrate with $\varepsilon_r = 16$ were calculated and are shown in Fig. 3, where the results of [2] and [4], which are nearly coincident, are also shown. The agreement is seen to be quite good.

To reveal the typical high-frequency performance, the characteristics of a slot-line configuration with symmetrically located shields $d_1 = d_2$ have been computed and the results are show in Figs. 4 and 5. The substrate is 1.0-mm-thick isotropic material with $\varepsilon_r = 9.6$ (Alumina). The results show two important characteristics. At high frequency, the effective dielectric constant tends to be independent of the line width, and the dispersion curves intersect with the characteristics of the dominant waveguide mode at a certain frequency, beyond which the guided propagation becomes impossible. At frequencies higher than this, the line radiates in the broadside directions. This frequency, which is independent of the slot width, limits the high-frequency response of the line. The slot-line is therefore usable over a range of frequencies limited by the cutoff at the low-frequency end and the high-frequency radiation limit.

Difficulties are encountered when the method is applied to slot-lines with narrow gaps $W/2d \ll 1$ and/or large distances of the shields $d_1/d > 10$ and $d_2/d > 10$. The computation time becomes excessive in both cases as the exponential factors do not decay sufficiently rapidly and therefore a larger number of equations is required to achieve reasonable accuracy. Very narrow slot-lines therefore may be calculated using other methods (e.g., Cohn's method). In case of large shield spacing, the method is still applicable taking into consideration the fact that at frequencies far from the cutoff the shields will have negligible effect on the line characteristics if the distances $d_1$ and $d_2$ are in excess of some minimum values $d_{1m}$ and $d_{2m}$. This is due to the exponential decay of the slow guided wave away from the plane of the line. If the actual distances are larger, then they can be set to these minimum distances without appreciable change in the computed characteristics which will be those of an open, unshielded line.

To obtain a rough estimate for $d_{1m}$ and $d_{2m}$, we note that from elementary considerations the effect of upper and lower shields will be negligible when

$$e^{-(4\pi/\lambda)d_1\sqrt{\varepsilon_{\text{eff}}-1}} \ll 1 \quad e^{-(4\pi/\lambda)d_2\sqrt{\varepsilon_{\text{eff}}-1}} \ll 1 \quad k_0 = \frac{2\pi}{\lambda}.$$

Then, $d_{1m}$ and $d_{2m}$ are the values of $d_1$ and $d_2$ which reasonable satisfy these inequalities. Practically, however, these values are checked by performing additional calculations at larger values of $d_1$ and $d_2$ making sure that the change is insignificant. Evidently, this is not applicable in the vicinity of the cutoff when $\varepsilon_{\text{eff}} \lesssim 1$. The results of Fig. 3 for the unshielded slot-line were obtained in this way.

## References


[1] S. B. Cohn, "Slot line on a dielectric substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, Oct. 1969.

[2] E. A. Mariani, C. P. Heinzman, J. P. Agrios, and S. B. Cohn, "Slot line characteristics," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, Dec. 1969.

[3] T. Itoh, and R. Mittra, "Dispersion characteristics of slot lines," *Electron. Lett.*, vol. 7, no. 13, July 1971.

[4] J. B. Knorr and K -D. Kuchler, "Analysis of coupled slots and coplanar strips on dielectric substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, July 1975.

[5] N. Samardzija and T. Itoh, "Double layered slot line for millimeter-wave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, Nov. 1976.

[6] H. Hofmann, "Dispersion of planar waveguides for millimeter-wave applications," *Arch. Elek. Ubertragung.*, vol. 31, no. 1, Jan. 1977.

[7] A -M. A. El-Sherbiny, "Exact analysis of shielded microstrip lines and bilateral fin-lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, July 1981.

[8] A -M. A. El-Sherbiny, "Hybrid mode analysis of microstrip lines on anisotropic substrates," presented at the IEEE MTT-S Intern. Symp. Los Angeles, June 15–19, 1981. An expanded version was published in *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, Dec. 1981.

[9] R. Mittra and S. W. Lee, *Analytical Techniques in the Theory of Guided Waves.* New York: McMillan, 1971.


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